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Nuclear fusion in charge-asymmetric muonic molecules

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Abstract. Nuclear fusion reactions in hydrogen-lithium muonic molecules, $h\mu^{6,7}$Li (where $h = p, d, t$) are considered and fusion rates from rotational states $J = 0$ of the molecules are presented. Results obtained depend on the isotopic composition of the molecules and range between $10^3\,\mathrm{s}^{-1}$ and $10^7\,\mathrm{s}^{-1}$. The upper limit for fusion rates from rotational states $J = 0$ of hydrogen-helium muonic molecules, $d\mu^{3,4}$He and $t\mu^{3,4}$He, equal $10^6\,\mathrm{s}^{-1}$, is also found.

PACS. 36.10.Dr Positronium, muonium, muonic atoms and molecules – 25.45.-z ^{2}H-induced reactions – 25.55.-e ^{3}H-, ^{3}He-, and ^{4}He-induced reactions

1 Introduction

Investigation of nuclear synthesis in charge-asymmetric muonic molecules, $h\mu Z$, ($h \equiv p, d, t$ is a hydrogen isotope, $Z \equiv$ He, Li is an isotope of helium or lithium nucleus, respectively) gives a possibility to investigate strong interaction at low energies [1]. Small energies, eV−keV, are not accessible in accelerator experiments and there is practically no information about strong interaction in such energy region. Properties of strong interaction as charge symmetry, iso-invariance and the character of P- and T-invariance have been established mainly in MeV and are not guaranteed in keV energy region.

The interest in such investigation is related also to the problem of the primordial nucleosynthesis of light nuclei in the early Universe [2]. For example, in stars and in the Galaxy one finds the deficiency of light nuclei (except for ^{4}He) as compared with the predictions based on the theory of thermonuclear reactions and generally accepted models. Deuterium, for example, disintegrates in stars at energies $\geq$ 50 eV, Li at $\geq$ 200 eV, and so on. At the same time the creation of light elements in keV-range of energies, *e.g.* in reactions $p + {}^7\mathrm{Li} \to 2\,{}^4\mathrm{He}$ and $t + {}^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma$ is also very important. To explain this phenomenon modified star models are usually proposed, which base on cross sections approximated from accelerator energy region to the astrophysical one, neglecting all possible resonances and other anomalies. It is not excluded, however, that nuclear cross sections have a resonance character, which leads to intensive burning of the light elements in stars. At the same time the cross sections for $h{-}Z$ fusion obtained by extrapolation of the astrophysical S-factor from high to low energies may be significantly underestimated as follows, *e.g.* from comparison of new experimental data for $d + d \to {}^4\mathrm{He}$ reaction for energy $<$ 100 keV with the ones extrapolated from higher energies.

In the present paper we consider only exothermal fusion reactions [3]. There are two possible fusion channels for $d{-}^3$He system

$$d\,^3\mathrm{He} \to \alpha + p + 18.3\ \mathrm{MeV} \quad (1)$$
$$\to {}^5\mathrm{Li} + \gamma + 16.4\ \mathrm{MeV}. \quad (2)$$

The radiative channel (2) is strongly suppressed as well as the fusion in $d\mu\,^4$He molecule [3,4]

$$d\,^4\mathrm{He} \to {}^6\mathrm{Li} + \gamma + 1.48\ \mathrm{MeV}. \quad (3)$$

Five fusion channels are possible for $t{-}^3$He system

$$t\,^3\mathrm{He} \to \alpha + p + n + 12.1\ \mathrm{MeV} \quad (4)$$
$$\to \alpha + d + 14.32\ \mathrm{MeV} \quad (5)$$
$$\to {}^5\mathrm{He} + p + 11.2\ \mathrm{MeV} \quad (6)$$
$$\to {}^5\mathrm{Li} + n + 10.13\ \mathrm{MeV} \quad (7)$$
$$\to {}^6\mathrm{Li} + \gamma + 15.8\,\mathrm{MeV} \quad (8)$$

and only one for $t{-}^4$He

$$t^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma + 2.47\,\mathrm{MeV}. \quad (9)$$

Numerous nuclear fusion reactions for $h{-}$Li system (see [3,4]) are presented in [5].

All fusion reactions mentioned above may occur in corresponding $h\mu$He and $h\mu$Li muonic molecules. Recently deexcitation and decay of such molecules (with rates

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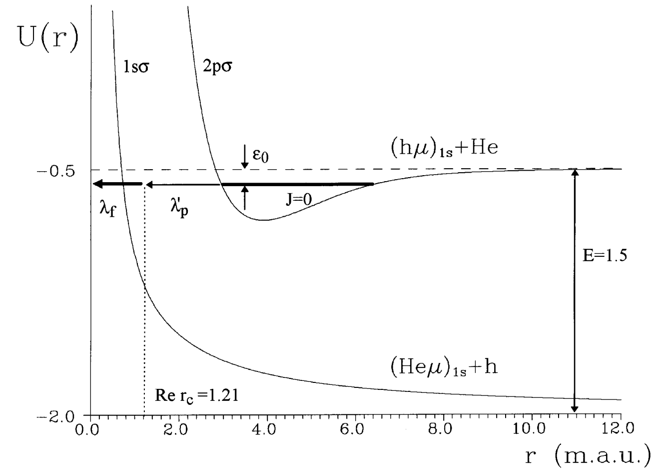

Fig. 1. Scheme of molecular terms for hydrogen-helium system. The corresponden asymptotic energies and transfer rates are indicated.

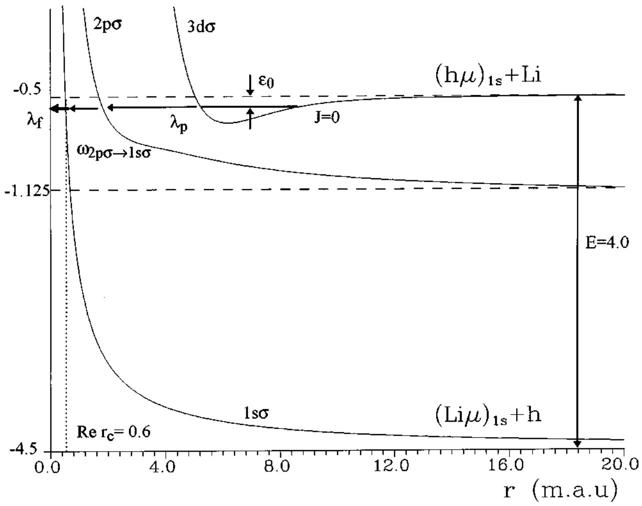

Fig. 2. Scheme of molecular terms for hydrogen-lithium system. The correspondent asymptotic energies and transfer rates are indicated.

$\geq 10^{11}\,\mathrm{s}^{-1}$) were considered in [6,7] and [8], respectively. As follows from results of [6,8–11] nonradiative decay of $p\mu$He and $p\mu$Li muonic molecules *via* predissociation prevail over radiative and Auger decay channels. Predissociation corresponds to transition between molecular terms $2p\sigma \to 1s\sigma$ for $h\mu$He (see Fig. 1 and *e.g.* [12]) and $3d\sigma \to 2p\sigma$ for $h\mu$Li (see Fig. 2 and *e.g.* [13]).

Because internuclear separation is relatively large in such molecules (about ~ 1000 fm) the nuclear reactions occur with a relatively small rates. Fusion rate for reaction (1) in $d\mu\,^3$He molecule $\lambda_f = 0.32 \times 10^6\,\mathrm{s}^{-1}$, estimated in [14], coincides in fact with the result $\lambda_f = 0.28 \times 10^6\,\mathrm{s}^{-1}$, obtained in [15] using more refined method for determination of the molecular state. At the same time it differs by one order of magnitude from result $\lambda_f = 3.8 \times 10^6\,\mathrm{s}^{-1}$ obtained in [16].

A possible enchancement of a fusion rate may be due to the presence of a threshold resonance, close to the threshold energy of some two-body channels, *e.g.* in reaction (5) ($\lambda_f = 0.58 \times 10^6\,\mathrm{s}^{-1}$ [1]), as well as in reaction

$$d\,^6\mathrm{Li} \to \,^8\mathrm{Be} + \gamma + 22.3\ \mathrm{MeV} \qquad (10)$$

that has $\lambda_f = 1.83\times10^9\,\mathrm{s}^{-1}$ calculated in [17]. The nuclei d and ^{6}Li in reaction (10) form the threshold resonant state $^8\mathrm{Be}^*(2^+,0)$ [17,18] as follows from Figure 2 and scheme of ^{8}Be levels [3] (this situation is analogous to nuclear fusion in $dt\mu$ molecule). Consequently, the value of the overlap integral characterizing the probability of a transition between the molecular and the nuclear resonance wave function is relatively large. Therefore, the long-range nuclear transition in the molecule can be expected.

Preliminary estimation of nuclear fusion rate in $h\mu$Li molecule obtained in [19] in the framework of one-level approximation for bound state in the $3d\sigma$-term resulted in a very small value $\sim 0.01\,\mathrm{s}^{-1}$. At the same time the upper limits for fusion rates λ_f ($< 10^{12}\,\mathrm{s}^{-1}$ for $p\mu$Li, $< 10^{10}\,\mathrm{s}^{-1}$ for $d\mu$Li and $< 10^9\,\mathrm{s}^{-1}$ for $t\mu$Li) were found in [20] under the assumption that nuclear fusion from the $1s\sigma$-state could provide the maximum rate of the fusion reaction. However it is necessary to calculate the contribution of this state into the total wave function of muonic molecule at small internuclear distances r. In order to do this one should solve a system of differential equations within a multilevel approximation. Determination of the wave function of $Z\mu h$ molecule for small r is not a simple problem because coefficients of the system are singular at $r \to 0$.

In the present paper reaction rates for nuclear fusion in $h\mu$Li and $h\mu$He muonic molecules from the rotational state $J = 0$ (where J is total angular momentum of the muonic molecule) are calculated using the semiclassical method. The influence of the possible quenching of the corresponding fusion rates (due to, *e.g.* selection rules or/and parity conservation) as well as its enhancement (due to presence of a nuclear resonance in the corresponding compound systems) are neglected.

2 Calculation methods and results

Methods presented in this section bases on the assumption that nuclear fusion in $Z\mu h$ molecule in $J = 0$ rotational state is more effective from the $1s\sigma$ molecular term due to relatively narrow potential barrier between nuclei and the lack of the centrifugal repulsion. So fusion reactions proceeded by the $2p\sigma \to 1s\sigma$ transition for $h\mu$He [14] and $3d\sigma \to 2p\sigma \to 1s\sigma$ transition for $h\mu$Li are considered.

The rate of nuclear fusion reaction for $h\mu$He was presented by equation (4) of [14] and for $h\mu$Li it can be written as

$$\lambda_f = \nu w_{3d\sigma\to2p\sigma} w_{2p\sigma\to1s\sigma} D, \qquad (11)$$

where ν is the frequency of nuclear oscillations in the molecule [8], $w_{3d\sigma\to2p\sigma}$ and $w_{2p\sigma\to1s\sigma}$ are the probabilities of the corresponding transitions. The barrier penetration factor is (muonic atom units, $\hbar = m = e = 1, m^{-1} = m_\mu^{-1} + M_h^{-1}$, are used unless otherwise indicated)

$$D = \exp(-2\int_{r_0}^{r_1} \sqrt{2M[U(r)-E]}dr). \qquad (12)$$

Here $U(r)$ is the effective potential corresponding to the $1s\sigma$-term including Coulomb repulsion of the nuclei, M is the reduced mass of the hydrogen and the lithium nuclei: $M^{-1} = M_h^{-1} + M_{\mathrm{Li}}^{-1}$, and E is the total energy of $h\mu$Li molecule in the $3d\sigma$ state (taking into account also the binding energy ε_0 of the $J = 0$ level) reckoned from the $r \to \infty$ limit of the $1s\sigma$ term (see Fig. 2)

$$E = -0.5 + 4.5 - \varepsilon_0. \qquad (13)$$

Keeping in mind smallness the of $\varepsilon_0 \sim 20$ eV [8] in comparison with the main term, 22.5 keV, we ignore ε_0 in (13).

The radius of the nuclear interaction, r_0, taken as a sum of the nuclear radii for the hydrogen and the lithium is $r_0 = 3$ fm for $p-$Li, and $r_0 = 4$ fm for $d-$Li and $t-$Li (as well as for $d-$He and $t-$He).

The r_1 in equation (12) is the turning point for evolution of the nuclei along $1s\sigma$-term with energy E ($r_1 = 0.68$ for $h\mu$He and $r_1 = 0.51$ for $h\mu$Li).

Predissociation rates $\lambda_p = \nu w_{3d\sigma\to 2p\sigma}$ for $h\mu$Li were calculated in [8][1]. So one can express equation (11) in the form

$$\lambda_f = \lambda_p w_{2p\sigma\to 1s\sigma} D. \qquad (14)$$

The $2s\sigma \to 1s\sigma$ transition probability can be written as

$$w_{2p\sigma\to 1s\sigma} = \exp(-2\delta), \qquad (15)$$

where δ is the Massey parameter

$$\delta = \mid Im \int_\kappa p(r)dr \mid . \qquad (16)$$

Here $p(r)$ is the radial momentum. The integration contour κ in the complex $r-$plane begins and ends at the turning points r_2 and r_1 of the $2p\sigma$ and $1s\sigma$ terms, respectively, and goes around the branch point r_c [21] of the terms. The $2p\sigma \to 1s\sigma$ transition is realized as subbarier transition *via* the complex branch point r_c in complex r-plane. We found[2] $r_c = 0.60 + 0.98i$ and the corresponding Massey parameter $\delta = 0.44\sqrt{2M}$.

The above method (referred to as the molecular predissociation method - MP) allows one to obtain common characteristics of fusion reactions in $h\mu$He and $h\mu$Li molecules such as D, $w_{2p\sigma\to 1s\sigma}$, λ_p (taken from [8]) and λ_f, presented in Table 1.

One can estimate nuclear fusion rates also by other method (referred to as the C-factor method – CF) that allows one to estimate the reaction rates in the separate fusion channels taking into account their peculiarities obtained *via* the constants of nuclear reactions C extracted from experimental data. This method results in λ_f close to that obtained by MP method when any quenching or enhancement of the fusion rate is absent. The effective cross section of nuclear fusion reaction for small collision energy (s-wave scattering) is given by [22]

$$\sigma = C\frac{\mid \psi(0) \mid^2}{v}, \qquad (17)$$

where C is the nuclear reaction constant (in $\mathrm{cm^3\,s^{-1}}$), $\psi(0)$ is the wave function describing the relative motion of the nuclei at $r = 0$ and v is the relative velocity of the nuclei at the infinity. After the $3d\sigma \to 2p\sigma \to 1s\sigma$ transition between molecular states with the total angular momentum $J = 0$ the $h\mu$Li system evolves along $1s\sigma$-term. The total angular momentum coincides with the relative angular momentum of the nuclei for $1s\sigma$-term as muon angular momentum is zero.

The wave function $\psi(r)$ was obtained as a result of the numerical solution of the Schrödinger equation for $J = 0$,

$$\chi''(r) + 2M[E - U(r)]\chi(r) = 0, \qquad (18)$$

where $\chi(r) = kr\psi(r)$ and $k = \sqrt{2ME}$ is the relative momentum of nuclei at infinity. The values $U(r)$ and E are determined in equation (12). The wave function $\chi(r)$ is normalized by the condition

$$\chi(r)_{r\to\infty} = \sin(kr + \delta_C - \xi \ln 2kr + \delta_0), \qquad (19)$$

where the Coulomb parameter $\xi = 2M/k$ corresponds to the repulsion of charges 1 and $Z - 1$ (taking into account the screening of the charge of the lithium nucleus by the muon), the Coulomb phase shift $\delta_C = \arg\Gamma(1 + i\xi)$ and δ_0 is the s-wave phase shift.

The nuclear fusion rate is given then by

$$\lambda_f = \sigma v n = C \mid \psi(0) \mid^2 n. \qquad (20)$$

Table 1. Predissociation rates λ_p [8], transition probabilities $w_{3d\sigma\to 2p\sigma}$ [8] and $w_{2p\sigma\to 1s\sigma}$ (Eq. (15)), penetration factors D (Eq. (12)), $\mid \psi(0) \mid^2$ (Eq. (18)), n, and fusion rates λ_f (Eq. (14)) obtained by MP method for $h-$Li. Fusion rates obtained by CF method are also presented for comparison.

	$p\mu\,^6$Li	$p\mu\,^7$Li	$d\mu\,^6$Li	$d\mu\,^7$Li	$t\mu\,^6$Li	$t\mu\,^7$Li
$\lambda_p(10^{12}\,\mathrm{s^{-1}})$	154	150	28.1	25.2	9.82	8.17
$w_{3d\sigma\to 2p\sigma}(10^{-3})$	7.74	7.39	1.59	1.41	0.60	0.49
$w_{2p\sigma\to 1s\sigma}(10^{-2})$	3.24	3.12	1.07	0.98	0.53	0.47
$D(10^{-7})$	122	109	4.64	3.55	0.492	0.325
$\mid \psi(0) \mid^2\ (10^{-7})$	207	183	4.34	3.22	0.364	0.230
$n(10^{26}\,\mathrm{cm^{-3}})$	162	149	10.99	8.9	2.04	1.55
$\lambda_f^{MP}(10^5\,\mathrm{s^{-1}})$	610	510	1.40	0.88	0.026	0.013
$\lambda_f^{CF}(10^5\,\mathrm{s^{-1}})$	5526	209	-	25	-	0.912

[1] They coincide with predissociation rates obtained in [17] by other method. The predissociation rates $\lambda_p' = \nu w_{2p\sigma\to 1s\sigma}$ for $h\mu$He were calculated in [6,9–11].

[2] The branch point for $3d\sigma \to 2p\sigma$ transition corresponds to $r_c = 4.02 + 1.56i$ [8].

The effective density of nuclei, n, is taken as $n = w_{3d\sigma\to 2p\sigma} w_{2p\sigma\to 1s\sigma} / \frac{4}{3}\pi R^3$ depending on the probability of the two-step transition and on the nuclear separation R immediately after the transition. We suppose that R coincides with the real part of the branch point r_c of $2p\sigma$ and $1s\sigma$ terms, *i.e.* $R = Rer_c$=0.6 for Liμh and 1.21 for Heμh [14]. Numerical values for $w_{3d\sigma\to 2p\sigma}$, $w_{2p\sigma\to 1s\sigma}$, $\mid \psi(0) \mid^2$ and n are presented in Table 1.

Conventionally the extrapolation of the cross sections for nuclear reactions to the low energy region (s-wave scattering) is performed in terms of the astrophysical $S(E)$-factor defined by [4]

$$\sigma = \frac{S(E)}{E} \exp(-2\pi\eta). \tag{21}$$

The factor $S(E)$ contains all information about nuclear interaction while a parameter $\eta = \alpha Zc/v$ (where $\alpha = 1/137$ is the fine structure constant, c is the light velocity) determines a Coulomb barrier penetration factor.

At the same time equation (21) could be written in the form (17) with $\mid \psi(0) \mid^2 = 2\pi\eta\ \exp(-2\pi\eta)$ [22], so

$$C = \frac{S(E)}{\pi\alpha Z M c} \cdot \tag{22}$$

In order to calculate fusion rate in a $h\mu$Li molecule we need the C-factor for $E = 22.5$ keV (Eq. (13)). Using $S = 68\ \mathrm{keV}\times b$ [23] for ^{7}Li$(p,\alpha)^4$He and data from Table 1 for $\mid \psi(0) \mid^2$ and n we obtained $C = 3.62 \times 10^{-17}\,\mathrm{cm^3\,s^{-1}}$ and

$$\lambda_f = 1 \times 10^7\,\mathrm{s^{-1}}. \tag{23}$$

The values of $\langle\sigma v\rangle$ for ^{7}Li$(p,\alpha)^4$He and ^{7}Li$(p,\alpha\gamma)\,^4$He averaged over the Maxwellian distribution of colliding nuclei were used for alternative determination of C from [4]:

$$C = \frac{\langle\sigma v\rangle}{\langle 2\pi\eta \exp(-2\pi\eta)\rangle} \cdot \tag{24}$$

We obtained the same C-factor for each final channel: $C = 3.88 \times 10^{-17}\,\mathrm{cm^3\,s^{-1}}$ for $T = 2\times 10^8$ K (*i.e.* $E \simeq 25$ keV). From equation (20) we have $\lambda_f = 1 \times 10^7\,\mathrm{s^{-1}}$ that is in a good agreement with result (23). So the total $\lambda_f = 2 \times 10^7\,\mathrm{s^{-1}}$ obtained by summing up two final channels is in agreement with λ_f^{MP} for $p\mu\,^7$Li from Table 1.

According to [4] the third possible channel ^{7}Li$(p,\gamma)\,^8$Be is suppressed.

For reaction ^{6}Li$(p,\alpha)\,^3$He we obtained $C = 1.67 \times 10^{-15}\,\mathrm{cm^3\,s^{-1}}$ [4][3] and $\lambda_f = 0.55 \times 10^9\,\mathrm{s^{-1}}$, that exceeds by one order of magnitude the corresponding value of λ_f^{MP} for $p\mu\,^6$Li.

For reaction ^{7}Li$(d,n)\,^8$Be we have $C = 0.88 \times 10^{-14}\,\mathrm{cm^3\,s^{-1}}$ [4] and $\lambda_f = 2.5 \times 10^6\ \mathrm{s^{-1}}$, that is about 25 times greater than the corresponding value of λ_f^{MP} for $d\mu\,^7$Li.

[3] The value $C = 1.71 \times 10^{-15}\,\mathrm{cm^3\,s^{-1}}$ was obtained from equation (22) with $S = 3.15$ MeV $\times\, b$ from [23].

Table 2. Nuclear fusion rates for $h-$He, λ_f^{CF} for $d\mu\,^3$He calculated for $R = Rer_c = 1.21$ [14] is also presented.

	$d\mu\,^3$He	$d\mu\,^4$He	$t\mu\,^3$He	$t\mu\,^4$He
$\lambda_p'(10^{11}\,\mathrm{s^{-1}})$	3.58	1.85	0.79	0.31
$D(10^{-7})$	24.8	-	4.61	-
$\mid \psi(0) \mid^2\ (10^{-7})$	22.2	21.1	9.7	1.09
$n(10^{26}\,\mathrm{cm^{-3}})$	0.39	0.2	0.094	0.064
$\lambda_f^{MP}(\mathrm{s^{-1}})$	8.8×10^5	-	3.6×10^4	-
$\lambda_f^{CF}(\mathrm{s^{-1}})$	3.2×10^5	6×10^{-5}	0.59×10^4	0.04

For reaction ^{7}Li$(t,2n)\,^8$Be we obtained $C = 2.18 \times 10^{-14}\,\mathrm{cm^3\,s^{-1}}$ [4] and $\lambda_f = 0.8\times 10^5\,\mathrm{s^{-1}}$, whereas for reaction ^{7}Li$(t,\alpha)\,^6$He we have from [3] $S = 14$ MeV$\times b$ and, according to equations (22) and (20) $C = 3.1\times 10^{-15}\,\mathrm{cm^3\,s^{-1}}$ and $\lambda_f = 1.1 \times 10^4\,\mathrm{s^{-1}}$, respectively. Suming the rates for these two channels we obtained $\lambda_f = 0.91 \times 10^5\,\mathrm{s^{-1}}$ that exceeds by two orders of magnitude the corresponding value of λ_f^{MP}.

The discrepancy between λ_f obtained by different methods (see Tab. 1) could be explained by any resonant enhancement which is not taken into account by MP method.

Below we consider $t-^3$He fusion reactions following our analysis in [14]. In order to estimate λ_f by MP method we calculated $D = 4.61 \times 10^{-7}$ and used $\lambda_p' = 0.79 \times 10^{11}\,\mathrm{s^{-1}}$ from [6]. So $\lambda_f = \lambda_p' D = 3.6 \times 10^4\,\mathrm{s^{-1}}$.

When calculating the rates by CF method we obtained the C-factor according to equation (24) for ^{3}He$(t,np)\,^4$He (reaction (4)) and ^{3}He$(t,d)\,^4$He (reaction (5)) using the data of [4] for $T = 10^8$ K corresponding to $E \simeq 10$ keV for $h\mu$He molecule [14]. We obtained $C = 3.8\times 10^{-16}\,\mathrm{cm^3\,s^{-1}}$ for reaction (4) and $C = 2.7 \times 10^{-16}\,\mathrm{cm^3\,s^{-1}}$ for reaction (5). Following equation (20) we calculated $\mid \psi(0) \mid^2 = 0.97 \times 10^{-6}$ and $n = w_{2p\sigma\to 1s\sigma}/\frac{4}{3}\pi R^3 = 0.09 \times 10^{26}\,\mathrm{cm^{-3}}$ using the transition probability from [6] and $R = Rer_c = 1.21$ from [14]. We obtained $\lambda_f = 0.35 \times 10^4\,\mathrm{s^{-1}}$ for reaction (4) and $\lambda_f = 0.24\times 10^4\,\mathrm{s^{-1}}$ for reaction (5). Summing up the fusion rates for both reactions (4, 5) we have $\lambda_f = 0.59 \times 10^4\,\mathrm{s^{-1}}$ that is close to the value obtained by MP method (see Tab. 2)[4].

As for muonic molecules with ^{4}He we obtained fusion rate for reaction (3) (discussed in [24]) by CF method. Using equation (20) with $\mid \psi(0) \mid^2 = 2.11 \times 10^{-6}$, $n = 0.2 \times 10^{26}\,\mathrm{cm^{-3}}$ and equation (22) with data taken from [4] for ^{4}He$(d,\gamma)\,^6$Li we obtained $C = 1.44\times 10^{-24}\,\mathrm{cm^3\,s^{-1}}$ and $\lambda_f = 0.6 \times 10^{-4}\,\mathrm{s^{-1}}$, that is to small to be measured experimentally.

[4] The alternative consideration of nuclear fusion in $t\mu\,^3$He molecule was presented in [1]. The authors obtained $\lambda_f = 5.8 \times 10^5\,\mathrm{s^{-1}}$ using resonant formation of the intermediate ^{6}Li$^*(3^+)$ nucleus from molecular state with $J = 2$. Our result corresponds to the nonresonant fusion from $J = 0$ state of the molecule formed at low collision energy.

We calculated also the rate for the analogous reaction (9) using equation (20). We obtained $|\psi(0)|^2 = 1.09 \times 10^{-7}$, $n = 0.06 \times 10^{26}\,\mathrm{cm}^{-3}$, and $C = 5.7 \times 10^{-20}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ from equation (22) with $S = 0.14$ keV $\times\, b$ for $^4\mathrm{He}(t,\gamma)\,^7\mathrm{Li}$ from [3]. The resulting fusion rate is $\lambda_f = 0.04\,\mathrm{s}^{-1}$.

Fusion rates in hydrogen-helium muonic molecules are collected in Table 2 together with λ_f obtained in [14] for $d\mu\,^3\mathrm{He}$.

The probability for fusion-in-flight in $h\mu + Z$ collision should be quenched (even in liquid targets) when compared with that from molecular state since the effective density of nuclei in the molecule, $n \sim 10^{25}$–$10^{28}\,\mathrm{cm}^{-3}$ (see Tabs. 1, 2), significantly exceeds the liquid hydrogen density $N = 4.25 \times 10^{22}\,\mathrm{cm}^{-3}$. Helium muonic molecules are more preferable than lithium ones [8,13] in experimental investigation of nuclear reaction due to their faster formation [25] and slower deexcitation [6,9–11]. However, numerous nuclear fusion reactions in $h\mu$Li molecules [3–5,23] could extend the possibilities of experimental investigation of nuclear fusion in charge-asymmetric muonic molecules. The methods of experimental investigation of nuclear fusion in $h\mu$He and $h\mu$Li are discussed in [5,24]. The preliminary experimental upper bounds for fusion rates for reaction (1) from $J = 0$ state of $d\mu\,^3\mathrm{He}$ molecule are presented in [26]. Namely $\lambda_f \leq 4 \times 10^7\,\mathrm{s}^{-1}$ for the rotational $1 \rightarrow 0$ transition rate λ_{10} taken from [7], and $\lambda_f \leq 1 \times 10^6\,\mathrm{s}^{-1}$ for λ_{10} taken from [15]. Our theoretical prediction lies below of these experimental upper bounds.

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